B.Sc. Part—I (Semester—II) (CBCS) Examination

MATHEMATICS

(Ordinary Differential Equations)

Paper—III

Time : Three Hours] [Maximum Marks : 60

Note : Question No. 1 is compulsory, attempt it once only.

1. Choose the correct alternative :

(i) A DE of the form $y' + Py = y^n Q$ is :

(a) Linear equation (b) Bernoulli's equation

(c) Clairaut's equation (d) None of these

(ii) A linear equation of the first order is of the form $y' + Py = Q$ in which ?

(a) P and Q are functions of x alone.

(b) P and Q are functions of y alone.

(c) P is a function of x and Q is a function of y.

(d) P is a function of y and Q is a function of x.

(iii) The differential equation of the orthogonal trajectories is obtained by replacing dy/dx by :

(a) –dy/dx (b) –dx/dy

(c) dx/dy (d) None of these

(iv) The form of the Clairaut's equation is :

(a) $y = x + f(p)$ (b) $y = x - f(p)$

(c) $y = px + f(p)$ (d) $y = px - f(p)$

(v) The roots of the equation $y'' - 4y' + y = 0$ are :

(a) real and distinct (b) Surds

(c) real and repeated (d) Complex

(vi) Particular integral of $\frac{1}{D+1}e^{2x}$ is :

(a) e^{2x} (b) $\frac{1}{2}e^{2x}$

(c) $\frac{1}{3}e^{2x}$ (d) $\frac{1}{5}e^{2x}$

(vii) If y_1 and y_2 are linearly dependent differentiable functions, then their wronskian :

(a) Vanishes (b) Never vanishes

(c) Vanishes identically (d) None of these

(viii) The particular solution of the DE $y'' + Py' + Qy = 0$ is $y = e^{-x}$ if :

(a) $P + xQ = 0$ (b) $1 + P + Q = 0$

(c) $1 + P - Q = 0$ (d) $1 - P + Q = 0$

(ix) A body originally at 80^0C cools down to 60^0 C in 20 minutes, the temperature of the surrounding being 40^0 C. The temperature x of the body after 40 minutes from the original will satisfy :

(a) $49 < x < 51$ (b) $51 < x < 53$

(c) $53 < x < 55$ (d) $55 < x < 57$

(x) The process of decay results into the emission of radiation and the element is called :

(a) Carbon dating (b) Ordinary carbon

(c) Steady state heat flow (d) Radioactive 10×1=10

UNIT—I

2. (a) Solve the DE $\cos x \, dy = y (\sin x - y) \, dx$. 6

OR

(b) Solve the DE $\cos^2 x \dfrac{dy}{dx} + y = \tan x$. 6

(c) Find the differential equation of a system of confocal ellipses $\dfrac{x^2}{a^2+\lambda} + \dfrac{y^2}{b^2+\lambda} = 1$, where λ is an arbitrary constant. 4

OR

(d) Show that the DE is exact and solve if exact $(x^2 - 4xy - 2y^2)\, dx + (y^2 - 4xy - 2x^2)\, dy = 0$. 4

UNIT—II

3. (a) Find the orthogonal trajectories of the family of coaxial circles $x^2 + y^2 + 2gx + c = 0$ where g is the parameter. 6

OR

(b) Solve $(p - xy)(p - x^2)(p - y^2) = 0$. 6

(c) Solve $p^2 + 2py \cot x = y^2$. 4

OR

(d) Explain Clairaut's equation and also find the primitive of $P = \log (px - y)$. 4

UNIT—III

4. (a) Solve $(D^3 - 3D^2 + 9D - 27)\, y = \cos 3x$. 6

OR

(b) Solve the DE $x^3\dfrac{d^3y}{dx^3} + 2x^2\dfrac{d^2y}{dx^2} + 2y = 10\left(x + \dfrac{1}{x}\right)$. 6

(c) Solve the equation $y'' - 4y' + y = 0$. 4

OR

(d) Solve $(D^2 - 4D + 13)^2\, y = 0$. 4

UNIT—IV

5. (a) Solve the variation of parameters $y'' + n^2 y = \operatorname{cosec} nx$. 6

OR

(b) Solve the DE by changing the independent variable $x^6\, y'' + 3x^5 y' + a^2 y = \dfrac{1}{x^2}$. 6

(c) Prove that the wronskian $w(y_1, y_2, x)$ of any two solutions y_1 and y_2 of the DE $y'' + py'$ $+ 94 = 0$, $p,q \in c^o$ satisfies the identity $w(y_1, y_2, x) = w(y_1, y_2, a)\, e^{-\int_a^x p(t)\,dt}$. 4

OR

(d) Solve the DE $(x \sin x + \cos x)\dfrac{d^2y}{dx^2} - x\cos x\dfrac{dy}{dx} + y\cos x = 0$. 4

UNIT—V

6. (a) A certain radioactive material is known to decay at a rate proportional to the amount present. If initially there is 10 mg of the material present and after 10 years it is observed that the material has lost 10 percent of its original mass, then find the amount of mass present at any time t. Also determine the mass of the material after 50 years. 6

OR

(b) How long will it take for the initial deposit to double at the rate 10 percent per year if the compounding is continuous ? 6

(c) A pipe 20 cm in diameter contains steam at 150^0C and is protected with a covering 5 cm thick for which K = 0.0025. If the temperature of the outer surface of the covering is 40^0C, find the temperature half way through the covering under steady state conditions. 4

OR

(d) According to Newton's law to cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temperature of the air is 30^0C and the substance cools from 100^0C to 70^0C in 15 minutes, find when the temperature will be 40^0C. 4